MA114 Summer II 2018
Worksheet 2a - Partial Fractions
6/11/18

Solutions

1. Write each of the following rational functions as a polynomial plus a proper rational function with a fully factored denominator.

a) $\dfrac{x-9}{x^2+3x-10} = \dfrac{x-9}{(x+5)(x-2)}$

b) $\dfrac{x^3+1}{x^2-4} = x + \dfrac{4x+1}{x^2-4} = x + \dfrac{4x+1}{(x+2)(x-2)}$

$$\begin{array}{r} x \\ x^2-4 \overline{)\,x^3+1} \\ -(x^3-4x) \\ \hline 4x+1 \end{array}$$ remainder

c) $\dfrac{2x+7}{2x^2+5}$ Already proper with fully factored denominator.

2. Compute the antiderivative $\int \frac{x^3+4}{x^2+4}\,dx$.

1) $x^2+4 \overline{)\, x^3+4}$ with x on top, $-(x^3+4x)$, remainder $-4x+4$, so $\int \frac{x^3+4}{x^2+4}dx = \int x + \frac{-4x+4}{x^2+4}dx$

$= \int \left(x - \frac{4x}{x^2+4} + \frac{4}{x^2+4}\right)dx$

- $\int \frac{-4x}{x^2+4}dx = -2\int \frac{1}{u}du = -2\ln|x^2+4|$

$= \boxed{\frac{1}{2}x^2 - 2\ln|x^2+4| + 2\arctan\left(\frac{x}{2}\right) + C}$

Let $u = x^2+4$

$du = 2x\,dx$

- $\int \frac{4}{x^2+4}dx = 4\cdot\frac{1}{2}\arctan\left(\frac{x}{2}\right)$

(arctan formula from class for u-sub $u=\frac{x}{2}$)

3. Given that

$$\frac{4x+1}{(x-2)(x+2)} = \frac{9/4}{x-2} + \frac{7/4}{x+2},$$

compute $\int \frac{x^3+1}{x^2-4}\,dx$. (Use 1b and the given equation.)

Using 1b), $\int \frac{x^3+1}{x^2-4}dx = \int \left(x + \frac{4x+1}{(x-2)(x+2)}\right)dx$.

By the given equation, this becomes $\int \left(x + \frac{9/4}{x-2} + \frac{7/4}{x+2}\right)dx = \frac{1}{2}x^2 + \frac{9}{4}\ln|x-2| + \frac{7}{4}\ln|x+2| + C$